Title :SIMILARITY ANALYSIS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS OF FLUID FLOW OF POROUS MEDIA

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The present thesis titled **"Similarity analysis of nonlinear partial differential equations of fluid flow of porous media"** is an outcome my researches on the study of the physics of the flow through porous media have become basic for many scientific and engineering applications, quite apart from the interest it holds for its purely scientific aspects. Such diversified fields as soil mechanics , ground water hydrology , petroleum engineering , water purification , industrial filtration , ceramic engineering , powder metallurgy , sanitary engineering , study of gas masks and the possibility of disposing atomic wastes by injection into depleted oil reservoirs , all rely heavily upon it as fundamental to their individual problem . The scope of hydromechanics in porous media allows for division into several parts such as fluids, the static's and the dynamics etc.

In petroleum reservoir engineering especially in the secondary recovery of oil process, the mathematical equations describing the physical behavior of the process under investigations are highly nonlinear partial differential equations that can be easily tackled and simplified by using the General Similarity Technique. The vigorous expansion of the mathematical study of non – linear problem dates from the sixties. The increased interest in the field of non- linear problems among the technician is partly due to the fact that mathematics has developed a powerful tool for the solution of such problems. The investigation of boundary value problems in non – linear differential equations is of central interest to the physicists, engineers and practitioners.

Lie's group theory of differential equations, which are more than hundred years old, has had a rebirth in the last twenty years, as people have begun to appreciate its virtues that it unifies the many adhoc methods known for solving differential equation and it provides powerful new ways to find the solution. This theory has application to both ordinary and partial differential equations and is not restricted to linear equation. Lie's showed that the order of an ordinary differential equation can be reduced by one constructively, if it is invariant under a one parameter Lie group of transformation. If a system of partial differential equations is invariant under a Lie group of transformation, one can find constructively, special solutions called the similarity solutions or invariant solutions, which remains unchanged under some subgroup of the full

group admitted by the system. Lie (1881) also indicated that for linear partial differential equations, invariance under a Lie group leads directly to superposition of solutions in terms of transformations.

The method to search particular type of transformations that transform the partial differential equation into ordinary differential equation is known as Similarity Method. The concept of mathematical similarity was first introduced by Helmholtz (1873) , through the dimensional analysis approach. Later on importance of similarity parameters in physical sciences was demonstrated by Reynolds (1900), Buckingham (1914), Blasius (1908) and Sedov (1959) and they developed the concept of similarity in a more meaningful way till Morgan (1952), Birkhoff (1950) showed that the boundary layer equations were invariant under certain group of transformations which reduced the number of independent variables by one. The power of various similarity techniques is recently demonstrated by various authors like Hansen (1964), Rogers and Ames (1989), Bluman and Kumei (1989).

The mathematical similarity, in spite of having the property of simplifying the analysis of physical situation is evident in rare cases in natural sciences. Today in the modern age, the use of similarity analysis is not only restricted to fluid mechanics or heat transfer, but it is much useful in studying physical, biological, medical, social sciences. These sciences use one of the fundamental properties of similarity transformations i.e. the collapse of multicurve data to a single curve or mathematical expression. It's an important media to tackle the present day problem of science.

Among the various similarity methods like separation of variable, free parameter, dimensional methods, group theoretic technique, etc., the group theoretic methods are more attractive because this method is not based on linear operators, superposition or any other requirements of linear solution technique and hence they are applicable to both linear and non-linear models.

A similarity representation is obtained for a boundary value problem if the governing differential equations and the associated boundary conditions are invariant under a group of transformations. However, if any of the equation and

boundary conditions is not invariant under a group, then problem becomes non-similar.

Unfortunately, texts and papers on which the present analysis is based on finding similarity solution of partial differential equations, seldom give reason and motivations for selecting transformations that reduced partial differential equations to ordinary differential equations. Moreover, examination of the standard treatises on applied mathematics is usually of little help in providing information on similarity analysis. In view of these facts, our aim is to outline systematic procedure for finding similarity solutions. In addition, various methods that have been evolved are reviewed and evaluated. But above remarks are not intended to imply that information on similarity analysis cannot be found in technical publications. In fact, extended treatment of similarity analysis and general similarity methods for solving certain class of differential equation are found in a number of reference. A summary of techniques for finding similarity solution of boundary value problem is very well explained by Hansen(1964), Ames(1965), Seshadri an Na(1985), Bluman and Kumei(1989), Drachner(1999).

The first chapter, gives the general introduction of the various stage of the development, scope of the present work and summary of the other chapters of the thesis. A physic – mathematical background is given that is necessary to understand the phenomenon given in the subsequent chapters

In second chapter, various methods of analysis that build on similarity concepts is presented and compared on the basis of the utility and the generality. We have extended successfully the different similarity techniques to cover the wide range of flow geometries. The General Similarity Technique of the Lie group of transformation is explained in detail.

In third chapter, we deal with a singular phase flow system in unsaturated porous media. The governing differential equations are partial differential equations. We have use lie group transformation to convert these equations into ordinary differential equation and exact solution of this equation is drive.

In fourth chapter, we have discussed the analytical solution of phenomena of instabilities in poly phase flow through homogeneous porous media with mean pressure. Here we have considered the fingers that occupied the average cross sectional area with help of statistical treatment of fingering. Here we have neglected the size and shape of individual fingers. The mathematical solution of non-linear differential system governing such fingering phenomena with mean capillary pressure is obtain using continuous group theoretic transformation technique of similarity method which transform the governing partial differential equation to ordinary differential equation. We have also discussed the existence and uniqueness of the solution in detail. The analytical solution of the derived ordinary differential equation is also discussed.

In fifth chapter, we have discusses the group invariance method for the flow of two immiscible fluids with different wettability through porous media. This type of phenomena of imbibition arises in poly phase flow of immiscible liquids through fractured porous medium due to difference in the wetting abilities of the phase. The application of infinitesimal group had leaves non-linear partial differential equation governing such flow invariant, further this invariant property given arise certain type of transformation that convert partial differential equation governing imbibition phenomena of homogenous porous media into ordinary differential equation which is certainly non-linear differential equation of boundary value type. The analytical solution is obtained and numerical solution is also obtained by Picard's method. Finally it is concluding that the total saturation in a medium is always remains unity and the saturation of both the immiscible liquids has the different path.

In sixth chapter, the phenomena of finger imbibition in malty phase flow through homogenous porous media with varying phase density and capillary pressure is discussed. This type of phenomenon usually arrives under certain conditions due to the simultaneous occurrence of two special phenomenon namely fingering and imbibition. The flow under the consideration is influence by injection of preferentially wetting less viscous fluid (water) into porous medium saturated with resident fluid (oil). The basic differential equation governing finger- imbibition are derived in analytical solution for average cross sectional area occupied by finger has been obtain by similarity method. The similarity equation which is an ordinary differential

equation is solved with the help of two parameter singular perturbation technique. It is worth to note that the idealize model consider in this chapter has great practical application in many hydrological situation.

In seventh chapter, the problem of ground water flow is analyzed by the method of two parameter singular perturbation with variable coefficient. The differential system governing the groundwater flow yields a second order liner differential equation in which the coefficient of first and second order derivative consists of all small parameter together with permeability factor. Here the constant term in the equation also contains a permeability factor. The entire problem is classified into two cases namely ground water flow in heterogeneous porous media on a sloping bedrock and flow in two layer soil with an inclined boundary when lower layer is heterogeneous and upper one homogeneous. The analytical solution of present problem is obtained by applying two parameter singular perturbation methods with variable coefficient. The convergence and the existence of the solution are also discussed. It is shown that when the boundary slope of the inclined is small and the flow rate is sufficiently small. The free surface of waterfalls is partly represented by stat line and also by arc of negative exponential curve.

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